AQA

AS
FURTHER MATHEMATICS
7366/2S

Paper 2 Statistics

Mark scheme

June 2024

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the 3rd answer.	1.1b	B1	1
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the 4th answer.	1.1b	B1	Expected B-Y
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Obtains correct z value **AWRT** 1.88 or correct t value **AWRT** 1.90 **PI**	1.1b	B1	$z = 1.8808$ $$68.2 \pm 1.8808 \times \sqrt{\frac{15.7}{120}}$$ (67.5, 68.9)
	Uses formula for upper or lower limit of a confidence interval using their z value or their t value. **PI**	1.1a	M1	
	Obtains correct confidence interval **AWRT** 67.5 and 68.9	1.1b	A1	
	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Uses correct formula for $E(Y)$ or $E(Y^2)$ **PI** by sight of 29.45 or 978.23	1.1a	M1	$E(Y) = 15 \times 0.16 + 21 \times 0.32 + 36 \times 0.29 + 43 \times 0.23$ $E(Y) = 29.45$
	Obtains correct value of $E(Y)$ or $E(Y^2)$ **oe** **PI** by correct variance.	1.1b	A1	$E(Y^2) = 15^2 \times 0.16 + 21^2 \times 0.32 + 36^2 \times 0.29 + 43^2 \times 0.23$ $E(Y^2) = 978.23$
	Uses correct formula for s or s^2 with their values for $E(Y)$ and $E(Y^2)$ or obtains $s^2 = 110.9275$	1.1a	M1	$s = \sqrt{978.23 - 29.45^2}$ $s = 10.532$
	Completes a reasoned argument to obtain $s = 10.53$ Must see $s = $ **AWRT** 10.532 before final rounded answer.	2.1	R1	$s = 10.53$ correct to two decimal places
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Obtains $m = 36$	1.1b	B1	$m = 36$
	Obtains the correct value of their $m - 1.5 \times 10.53$ They may use a more accurate value of s	1.1a	M1	$P(Y > 36 - 1.5 \times 10.53)$ $= P(Y > 20.2)$ $= P(Y \geq 21)$
	Obtains 0.84 **CSO**	1.1b	A1	$= 0.32 + 0.29 + 0.23$ $= 0.84$
	Subtotal		**3**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(i)	States discrete uniform distribution. Condone omission of 'discrete' and 'distribution'.	3.3	B1	Discrete uniform distribution
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(ii)	Obtains $\dfrac{3}{8}$ **oe**	3.4	B1	$\dfrac{3}{8}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(i)	Explains that we would expect the frequencies/relative frequencies to be close to each other or the expected frequencies are 125 or the expected relative frequencies are 0.125 for a discrete uniform distribution.	2.4	E1	Using the model, we would expect the frequencies to be close to each other. However, the data shows a much higher frequency for landing on 5
	Recognises that observed frequencies/relative frequencies are not approximately equal or frequencies are not close to 125 or relative frequencies are not close to 0.125. Do not accept "not the same". Allow specific examples where different frequencies are compared Comments must relate to the data so there is evidence of bias alone is not sufficient but accept e.g. biased towards 5 Note that comments such as observed frequencies are not similar enough for a discrete uniform distribution scores both marks.	3.5b	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)(ii)	Suggests calculating the relative frequencies. Condone calculating probabilities using Clare's results.	3.5c	E1	Calculate the relative frequencies using Clare's results. Use the relative frequencies as probabilities for a discrete random variable.
	Suggests that the probabilities from the discrete uniform distribution are replaced with the relative frequencies or the discrete uniform distribution is replaced by a discrete random variable. Note that comments such as change the probabilities to relative frequencies scores both marks.	3.5c	E1	
	Subtotal		**2**	

| | **Question total** | | **6** | |

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Uses $\int_{2}^{5}\left(\dfrac{3x}{44}+\dfrac{1}{22}\right)dx$ or $\int_{1}^{2}\left(\dfrac{3x}{44}+\dfrac{1}{22}\right)dx$ Condone missing dx **PI** by sight of **AWRT** 0.852 or 0.148	1.1a	M1	$P(X>2)=\int_{2}^{5}\dfrac{3x}{44}+\dfrac{1}{22}\,dx$ $=\dfrac{75}{88}$
	Obtains $\dfrac{75}{88}$ **AWRT** 0.852	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Integrates a multiple of $\dfrac{3x}{44}+\dfrac{1}{22}$ to an expression of the form $ax^2 + bx$	1.1a	M1	Let q be the value of the upper quartile $$\int_{1}^{q}\frac{3x}{44}+\frac{1}{22}\ \mathrm{d}x = \frac{3}{4}$$ $$\left[\frac{3x^2}{88}+\frac{x}{22}\right]_{1}^{q} = \frac{3}{4}$$ $$\frac{3q^2}{88}+\frac{q}{22}-\frac{3}{88}-\frac{1}{22}=\frac{3}{4}$$ $$\frac{3q^2}{88}+\frac{q}{22}-\frac{7}{88}=\frac{3}{4}$$ $$\frac{3q^2}{88}+\frac{q}{22}-\frac{73}{88}=0$$ $$q = 4.31$$
	Integrates $k\left(\dfrac{3x}{44}+\dfrac{1}{22}\right)$ to obtain $k\left(\dfrac{3x^2}{88}+\dfrac{x}{22}\right)$	1.1b	A1	
	Substitutes the limits q and 1 into their integral of $\dfrac{3x}{44}+\dfrac{1}{22}$, subtracts and sets equal to $\dfrac{3}{4}$ to form a quadratic equation in q $$\frac{3q^2}{88}+\frac{q}{22}-\frac{3}{88}-\frac{1}{22}=\frac{3}{4}\ \textbf{oe}$$ or the limits 5 and q into their integral of $\dfrac{3x}{44}+\dfrac{1}{22}$, subtracts and sets equal to $\dfrac{1}{4}$ to form a quadratic equation in q $$\frac{75}{88}+\frac{5}{22}-\frac{3q^2}{88}-\frac{q}{22}=\frac{1}{4}\ \textbf{oe}$$	1.1a	M1	
	Obtains **AWRT** 4.31 If –5.64 is found it must be rejected	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Uses correct formula for $k\mathrm{E}(X^{-3})$ **PI** Condone mislabelling	1.1a	M1	$\mathrm{E}(44X^{-3}) = \int_{1}^{5} 3x^{-2} + 2x^{-3} \ \mathrm{d}x$ $\mathrm{E}(44X^{-3}) = \dfrac{84}{25}$
	Uses correct formula for $k\mathrm{E}(X^{-6})$ **PI** Condone mislabelling	1.1a	M1	$\mathrm{E}(44^{2}X^{-6}) = \int_{1}^{5} 132x^{-5} + 88x^{-6} \ \mathrm{d}x$ $\mathrm{E}(44^{2}X^{-6}) = 50.541568$
	Obtains correct value of $\mathrm{E}(X^{-3}) = \dfrac{21}{275}$ **oe** or **AWRT** 0.076 or $\mathrm{E}(44X^{-3}) = \dfrac{84}{25}$ **oe** and $\mathrm{E}(X^{-6})$ **AWRT** 0.026 or $\mathrm{E}(44^{2}X^{-6})$ **AWRT** 50.54 **PI**	1.1b	A1	$\mathrm{Var}(44X^{-3}) = 50.541568 - \left(\dfrac{84}{25}\right)^{2}$ $\mathrm{Var}(44X^{-3}) = 39.252$
	Uses $\mathrm{Var}(44X^{-3})$ $\quad = \mathrm{E}(44^{2}X^{-6}) - (\mathrm{E}(44X^{-3}))^{2}$ **oe** **PI** Condone mislabelling	1.1a	M1	
	Obtains **AWRT** 39.252	1.1b	A1	
	Subtotal		**5**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	States both hypotheses using correct notation. If a letter other than λ or μ is used, it must be correctly defined	2.5	B1	$H_0 : \lambda = 6$ $H_1 : \lambda \neq 6$ $X \sim \text{Po}(6)$
	Uses Poisson model $X \sim \text{Po}(6)$ to calculate one of $P(X \geq 11)$ **AWRT** 0.04, $P(X \leq 10)$ **AWRT** 0.96, $P(X \leq 11)$ **AWRT** 0.98 or $P(X > 11)$ **AWRT** 0.02, **PI** by correct upper tail of the critical region Condone mislabelling the probabilities	3.3	M1	$P(X \geq 11) = 0.043$ $0.043 > 0.025$ Do not reject H_0 There is not sufficient evidence to suggest that the mean number of customers per hour has changed.
	Uses Poisson model to calculate $P(X \geq 11)$ **AWRT** 0.043 or obtains the upper tail of the critical region $X \geq 12$ Condone mislabelling the probability	3.4	A1	
	Evaluates the Poisson model by correctly comparing their probability with 0.025 or 0.05 if a one-tailed test is performed or by correctly comparing 11 with their upper tail of the critical region	3.5a	M1	
	Infers H_0 not rejected. **FT** comparison of their $P(X \geq 11)$ or $P(X > 11)$ with 0.025 or 0.05 if a one-tailed test is performed Condone Accept H_0 or Reject H_1	2.2b	A1F	
	Concludes, from a fully correct comparison, in context by referring to a change in the mean (number of) customers per hour. **oe** (Conclusion must not be definite, eg use of 'suggest', 'support')	3.2a	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Uses Poisson model to calculate one of P($X \leq 1$), P($X \leq 2$), P($X \leq 10$), P($X \leq 11$), P($X \geq 11$) or P($X \geq 12$) Condone mislabelling the probabilities	3.4	M1	P($X \leq 1$) = 0.01735 P($X \leq 2$) = 0.06197 P($X \geq 11$) = 1 – P($X \leq 10$) P($X \geq 11$) = 1 – 0.95738
	Obtains P($X \leq 1$) = **AWRT** 0.017, and P($X \leq 11$) = **AWRT** 0.98 or P($X \geq 12$) = **AWRT** 0.02 Condone mislabelling the probabilities	1.1b	A1	P($X \geq 11$) = 0.04262 P($X \geq 12$) = 1 – P($X \leq 11$) P($X \geq 12$) = 1 – 0.97991 P($X \geq 12$) = 0.02009
	Adds the probability of their lower tail to the probability of their upper tail.	1.1a	M1	Probability of Type I error = 0.01735 + 0.02009 = 0.0374
	Completes a reasoned argument to obtain probability of Type I error. **AWRT** 0.0374 The values of the following probabilities with correct labelling must be seen: P($X \leq 1$), P($X \leq 2$), P($X \geq 11$) and P($X \geq 12$) or P($X \leq 1$), P($X \leq 2$), P($X \leq 10$) and P($X \leq 11$)	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Explains that the Poisson condition of independence does not hold.	3.5b	E1	Some customers are not entering the store independently.
	Subtotal		**1**	

	Question total		**11**	

	Paper total		**40**	